牛客网暑期ACM多校训练营（第一场）

一. 编程题

1. Count the number of n x m matrices A satisfying the following condition modulo (10^9+7).
* $A_{i,j} \in \{0, 1, 2\}$ for all $1 \le i \le n, 1 \le j \le m$.
* $A_{i,j} \le A_{i+1,j}$ for all $1 \le i < n, 1 \le j \le m$.
* $A_{i,j} \le A_{i,j+1}$ for all $1 \le i \le n, 1 \le j < m$.

输入描述：
The input consists of several test cases and is terminated by end-of-file.
Each test case contains two integers n and m.
输出描述：
For each test case, print an integer which denotes the result.
备注
* $1 \le n, m \le 10^3$
* The number of test cases does not exceed 10^5.

示例1:
输入
1 2
2 2
1000 1000
输出
6
20
540949876

正确答案：

2. Count the number of n x n matrices A satisfying the following condition modulo m.
* $A_{i,j} \in \{0, 1, 2\}$ for all $1 \le i, j \le n$.
* $A_{i,j} = A_{j,i}$ for all $1 \le i, j \le n$.
* $A_{i,1} + A_{i,2} + ... + A_{i,n} = 2$ for all $1 \le i \le n$.
* $A_{1,1} = A_{2,2} = ... = A_{n,n} = 0$.
输入描述：
The input consists of several test cases and is terminated by end-of-file.
Each test case contains two integers n and m.
输出描述：
For each test case, print an integer which denotes the result.
备注
* $1 \le n \le 10^5$
* $1 \le m \le 10^9$
* The sum of n does not exceed 10^7.
示例1:
输入

3 1000000000
100000 1000000000
输出
1
507109376

正确答案：

3. In ICPCCamp, there are n switches and m bulbs.The m bulbs are ON at the beginning.
Bobo knows in advance an n x m binary matrix $C_{i,j}$. When the i-th switch is pressed, all the bulbs j satisfying $C_{i,j} = 1$ flip its state between ON and OFF.
Let f(S) be the number of bulbs staying ON after the switches in set S is pressed. Find the sum of $f(S)^3$ (cubic of f(S)) modulo (10^9+7) over all 2^n possible choices of S.
输入描述：
The input consists of several test cases and is terminated by end-of-file.
The first line of each test case contains two integers n and m.
The i-th of the following n lines contains m integers $C_{i,1}, C_{i,2}, ..., C_{i,m}$.
输出描述：
For each test case, print an integer which denotes the result.
备注
* $1 \le n \le 50$
* $1 \le m \le 1000$
* $C_{i,j} \in \{0, 1\}$
* The number of test cases does not exceed 500.
* The number of test cases with m > 50 does not exceed 1.
示例1:
输入
2 2
01
10
2 3
110
011
输出
10
30
说明
For the first sample, there are $2^2 = 4$ choices of S.
* $S = \emptyset$, f(S) = 2, $f(S)^3 = 8$.
* S = {1}, f(S) = 1, $f(S)^3 = 1$.
* S = {2}, f(S) = 1, $f(S)^3 = 1$.
* S = {1, 2}, f(S) = 0, $f(S)^3 = 0$.
Thus, the result is 8 + 1 + 1 + 0 = 10.

正确答案：

4. Two undirected simple graphs $G_1 = \langle V, E_1 \rangle$ and $G_2 = \langle V, E_2 \rangle$ where $V = \{1, 2, \ldots, n\}$ are isomorphic

when there exists a bijection ϕ on V satisfying $\{\phi(x), \phi(y)\} \in E_1$ if and only if {x, y} ∈ E_2.
Given two graphs $G_1 = \langle V, E_1 \rangle$ and $G_2 = \langle V, E_2 \rangle$, count the number of graphs $G = \langle V, E \rangle$ satisfying the following condition:
* $E \subseteq E_2$.
* G_1 and G are isomorphic.

输入描述：
The input consists of several test cases and is terminated by end-of-file.
The first line of each test case contains three integers n, m_1 and m_2 where $|E_1| = m_1$ and $|E_2| = m_2$.
The i-th of the following m_1 lines contains 2 integers a_i and b_i which denote $\{a_i, b_i\} \in E_1$.
The i-th of the last m_2 lines contains 2 integers a_i and b_i which denote $\{a_i, b_i\} \in E_2$.

输出描述：
For each test case, print an integer which denotes the result.

备注
* 1 ≤ n ≤ 8
* $1 \le m_1 \le m_2 \le \frac{n(n-1)}{2}$
* 1 ≤ a_i, b_i ≤ n
* The number of test cases does not exceed 50.

示例1:
输入
3 1 2
1 3
1 2
2 3
4 2 3
1 2
1 3
4 1
4 2
4 3
输出
2
3

正确答案：

5. Bobo has a sequence of integers $s_1, s_2, ..., s_n$ where 1 ≤ s_i ≤ k.
Find out the number of distinct sequences modulo (10^9+7) after removing exactly m elements.

输入描述：
The input consists of several test cases and is terminated by end-of-file.
The first line of each test case contains three integers n, m and k.
The second line contains n integers $s_1, s_2, ..., s_n$.

输出描述：
For each test case, print an integer which denotes the result.

备注
* 1 ≤ n ≤ 10^5
* 1 ≤ m ≤ min{n - 1, 10}
* 1 ≤ k ≤ 10

* $1 \le s_i \le k$
* The sum of n does not exceed 10^6.

示例1:
输入
3 2 2
1 2 1
4 2 2
1 2 1 2
输出
2
4

正确答案：

6.
Given $a_1, a_2, ..., a_n$, find

$$\sum_{x_1=1}^{a_1}\sum_{x_2=1}^{a_2}\cdots\sum_{x_n=1}^{a_n}\max\{x_1, x_2, \ldots, x_n\}$$

modulo (10^9+7).

输入描述：
The input consists of several test cases and is terminated by end-of-file.
The first line of each test case contains an integer n.
The second line contains n integers $a_1, a_2, ..., a_n$.
输出描述：
For each test case, print an integer which denotes the result.
备注
* $1 \le n \le 1000$
* $1 \le a_i \le 10^9$
* The number of test cases does not exceed 10.

示例1:
输入
2
1 2
5
2 3 3 3 3

输出
3
453

正确答案：

7. Bobo has a connected undirected simple graph with n vertices and m edges. The vertices are numbered by 1, 2, ..., n conveniently.
Given an integer k, Bobo chooses a subset of edges such that the vertices 1, 2, ..., k can reach each other via the chosen edges. He wants the chosen number of edges is minimized. Find out the number of ways to

choose modulo (10^9+7).

输入描述：
The input consists of several test cases and is terminated by end-of-file.
The first line of each test case contains three integers n, m and k.
The i-th of the following m lines contains 2 integers a_i and b_i which denote the edge between vertices a_i and b_i.
输出描述：
For each test case, print an integer which denotes the result.
备注
* 1 ≤ n ≤ 50
* $n-1 \le m \le \min\{\frac{n(n-1)}{2}, 1000\}$
* $1 \le k \le \min\{n, 12\}$
* 1 ≤ a_i, b_i ≤ n
* The number of test cases does not exceed 100.
* The number of test cases with n > 8 does not exceed 5.
示例1:
输入
4 4 2
1 3
1 4
2 3
2 4
4 3 3
1 4
2 4
3 4
3 2 2
1 3
2 3
输出
2
1
1

正确答案：

8. In ICPCCamp, there are n cities numbered with 1, 2, ..., n and (n - 1) bidirectional roads. The i-th road has color c_i and connects cities a_i and b_i. It is guaranteed that cities can reach each other.
Going from city u to city v via the unique shortest path, one may pass roads e_1, e_2, ..., e_k in order. Bobo denotes $(c_{e_2}-c_{e_1})^2+\cdots+(c_{e_k}-c_{e_{k-1}})^2$ as d(u, v).
For each city u, find out $f_u=\max_v d(u,v)$.
输入描述：
The input consists of several test cases and is terminated by end-of-file.
The first line of each test case contains an integer n.
The i-th of the following (n - 1) lines contains 3 integers a_i, b_i, c_i.

输出描述：
For each test case, print n integers $f_1, f_2, ..., f_n$.
备注
* $1 \le n \le 10^5$
* $1 \le a_i, b_i \le n$
* $1 \le c_i \le 10^5$
* The sum of n does not exceed 10^6.
示例1:
输入
2
1 2 1
4
1 2 1
2 3 2
3 4 1
输出
0
0
2
1
1
2

正确答案：

9. Two strings $u_1\ u_2\ ...\ u_k$ and $v_1\ v_2\ ...\ v_l$ are isomorphic if and only if k = l and there exists a injection g such that $u_i = g(v_i)$ for all $i \in \{1, 2, ..., k\}$.
Note that a function f is injection if and only if $f(x) \neq f(y)$ for all $x \neq y$.
Bobo would like to choose some strings from all n(n +1)/2 substrings of the given string $s_1\ s_2\ ...\ s_n$.
Find out the maximum number of strings he may choose so that no two chosen strings are isomorphic.
输入描述：
The input consists of several test cases and is terminated by end-of-file.
The first line of each test case contains an integer n.
The second line contains a string $s_1, s_2, ..., s_n$.
输出描述：
For each test case, print an integer which denotes the result.
备注
* $1 \le n \le 5x\ 10^4$
* $s_i \in \{a, b, c\}$
* The sum of n does not exceed $2 x 10^5$.
示例1:
输入
4
abaa
4
abab
输出

6
4

正确答案：

10. Given a sequence of integers a_1, a_2, ..., a_n and q pairs of integers (l_1, r_1), (l_2, r_2), ..., (l_q, r_q), find count(l_1, r_1), count(l_2, r_2), ..., count(l_q, r_q) where count(i, j) is the number of different integers among a_1, a_2, ..., a_i, a_j, a_{j+1}, ..., a_n.

输入描述：

The input consists of several test cases and is terminated by end-of-file.

The first line of each test cases contains two integers n and q.

The second line contains n integers a_1, a_2, ..., a_n.

The i-th of the following q lines contains two integers l_i and r_i.

输出描述：

For each test case, print q integers which denote the result.

备注

* $1 \le n, q \le 10^5$
* $1 \le a_i \le n$
* $1 \le l_i, r_i \le n$
* The number of test cases does not exceed 10.

示例1:

输入

3 2
1 2 1
1 2
1 3
4 1
1 2 3 4
1 3

输出

2
1
3

正确答案：