ICPC Asia::Tokyo 2014 Problem J – Exhibition

J: Exhibition – Solution (1/3)

Objective function can be written as follows:

$$\text{find} \quad \min_{\alpha,\beta,\gamma\in[0,1]} \alpha A + \beta B + \gamma C$$

$$\text{such that} \quad \min_{S\subseteq\{1,2,\ldots,n\},|S|=k-1} ((1-\alpha)x_1 + \sum_{i\in S} x_i) \times ((1-\beta)y_1 + \sum_{i\in S} y_i) \times ((1-\gamma)z_1 + \sum_{i\in S} z_i)$$

$$\leq \min_{S\subseteq\{2,3,\ldots,n\},|S|=k} (\sum_{i\in S} x_i) \times (\sum_{i\in S} y_i) \times (\sum_{i\in S} z_i)$$

★ min cost when we choose product 1

★ min cost when we do not choose product 1

(★) does not depend on α,β,γ. How to compute it?

J: Exhibition – Solution (2/3)

$$\star \quad \min_{S \subseteq \{2,3,\ldots,n\}, |S|=k} (\sum_{i \in S} x_i) \times (\sum_{i \in S} y_i) \times (\sum_{i \in S} z_i)$$

- Let F be combin(n-1, k) points in the XYZ-space consisting of ($\Sigma_{i \in S} x_i$, $\Sigma_{i \in S} y_i$, $\Sigma_{i \in S} z_i$) for each S.
- Minimizer of (★) is a vertex of convex hull of F.
- Let S^* be minimizer. Then there exists $p,q \in (0,1)$ with $p+q < 1$ s.t.

$$S^* = \arg\min_S p(\sum_{i \in S} x_i) + q(\sum_{i \in S} y_i) + (1-p-q)(\sum_{i \in S} z_i)$$

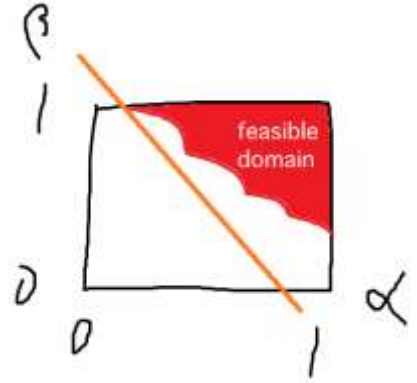
- If we know p,q, ↑ can be minimized by sorting.
- There are at most $O(n^4)$ candidates for such p,q. So we can compute (★) in $O(n^5 \log n)$ time.

J: Exhibition – Solution (3/3)

$$\min_{S\subseteq\{1,2,\ldots,n\},|S|=k-1}((1-\alpha)x_1+\sum_{i\in S}x_i)\times((1-\beta)y_1+\sum_{i\in S}y_i)\times((1-\gamma)z_1+\sum_{i\in S}z_i) \quad (\bigstar)$$

- For fixed α,β,γ, (★) can be minimized in the similar manner. (We consider at most $O(n^4)$ ways for S.)
- Feasible domain is complement of a union of convex sets..

$$\text{find} \min_{\alpha,\beta,\gamma\in[0,1]} \alpha A+\beta B+\gamma C$$



- Minimizer is achieved at the edge of cube $[0, 1]^3$. Try all of edges and take the best one.

J: Exhibition – Summary

Acceptance:

- !#$%&()*+-./:;<=>?@[\]^_`{|}~ (The University of Tokyo)
- H2O (Peking University)